MATHEMATICS
EXPLORATION PROBLEMS

Name : _____ **Index Number :** _____

Country : _____



19th International Mathematics and Science Olympiad
Indonesia
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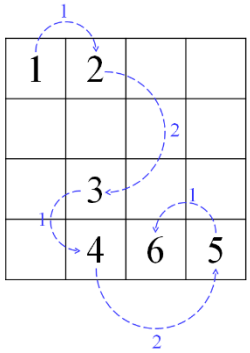
Instructions:

1. Write your name, country and index number on every page of the Answer Sheet.
2. Write your answers only in the Answer Sheet.
3. Answer all questions in Arabic Numerals or English.
4. There are 6 questions in this paper.
5. Each question is worth 6 marks and partial credit may be awarded.
6. You have 120 minutes to complete this paper.
7. You are provided with some manipulatives for exploration on some questions.
8. Use black pen or blue pen or pencil to write your answer.

Do not turn over this page until you are told to do so.

EXPLORATION PROBLEMS

1. In the diagram below, there is a 4 × 4 grid with a chain of consecutive positive integers starting from 1; i.e. 1 → 2 → 3 → 4 → 5 → 6, where the distances towards the next number are 1, 2, 1, 2, 1 sequentially (1 and 2 alternately).



Construct a chain of consecutive positive integers **as long as possible** in both grids below, starting from 1 at the top left corner and the distances towards the next number being 1, 2, 1, 2, … in the alternating manner.

What is maximum number that can be placed on a:

(i) 4 x 4 grid? Draw one example.

(ii) 5 x 5 grid? Draw one example.

1			

(a) 4 × 4 table
(2 points)

1				

(b) 5 × 5 table
(4 points)

2. Construct five different rectangles of various sizes by selecting each of the numbers from 1, 2, 3, 4, 5, 6, 7, 8, 9 or 10 exactly once to be the length and width dimensions. By using these five rectangles, can these pieces be tiled (without overlapping) to form:

(I) a 11 x 11 square? If yes, draw one such example. If not, state the reason why it is not possible.

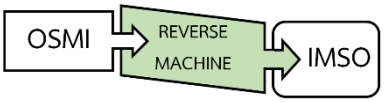
(II) a 13 x 13 square? If yes, draw one such example. If not, state the reason why it is not possible.

Note that all rectangles formed can be rotated or reflected.

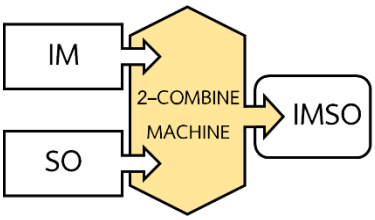
3. In the first move, we can make a straight cut across a square paper, dividing it into two smaller polygons. In each subsequent move, we can divide any one of the resulting polygons into two with a straight cut. No folding while cutting and no two papers can cut together.
   a. What is the minimum number of cuts required to obtain 2 polygons each with exactly 20 sides, along with some other polygons?
   b. What is the minimum number of cuts required to obtain 3 polygons each with exactly 30 sides, along with some other polygons?
   c. What is the minimum number of cuts required to obtain 4 polygons each with exactly 60 sides, along with some other polygons?

4. We have three kinds of machines namely “REVERSE”, “2-COMBINE” and “3-COMBINE” Machines.

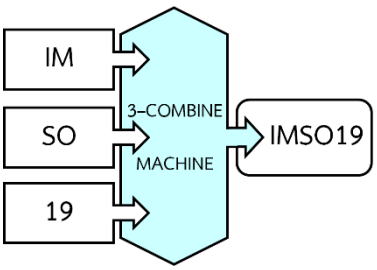
   A “REVERSE MACHINE” will reverse the order of the letters of the input as shown below:



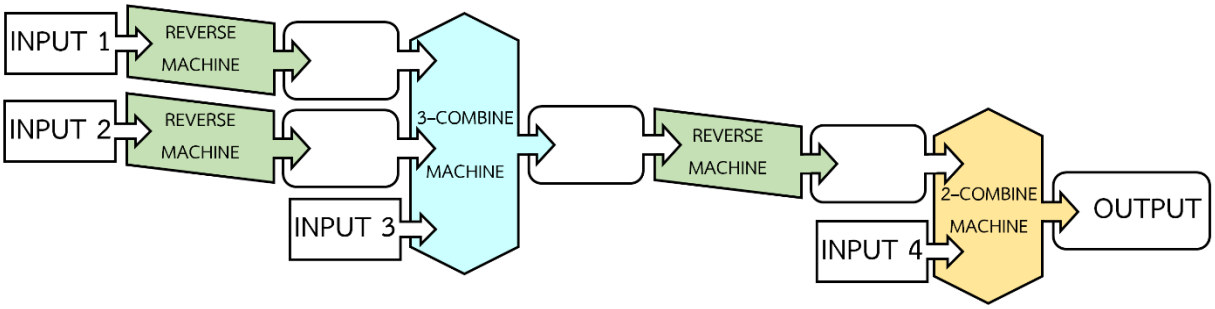
   A “2–COMBINE MACHINE” will connect 2 inputs together (putting the upper input in front of the lower input) as shown below:



   A “3–COMBINE MACHINE” will connect 3 inputs together (putting the upper-most input in front of the middle input and then followed by the lower-most input) as shown below:



   The following system consists of three “REVERSE MACHINES”, one “2–COMBINE MACHINE” and one “3–COMBINE MACHINE”.



   a. Give TWO (2) examples for “INPUT 1”, “INPUT 2”, “INPUT 3” and “INPUT 4” such that the “OUTPUT” will be “ILOVEIMSO19”.

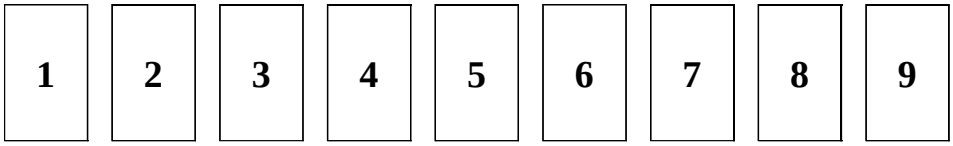
   b. How many different arrangements can we have for “INPUT 1”, “INPUT 2”, “INPUT 3” and “INPUT 4” such that the “OUTPUT” will be “ILOVEIMSO19”? (Note that each input must have at least 1 letter.)

5. Given a sequence such that each number only contains odd digits:

   1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 31, 33, 35, 37, 39, 51, 53, 55, …

   a) Find the 2022^{nd} term.
   b) Find the smallest term such that its sum of digits is 2022.
   c) Find the sum of all terms that does not exceed 2022.

6. Alam has 9 cards, where in each card, the numbers from 1, 2, 3, …, 9 are written.



   At first, he takes one odd and one even numbered card. Afterwards, he again takes one odd and one even numbered card from the remaining cards. Now, he has four cards and then arranges them into a row. These four cards should have the following properties:

   1) The product of each and every pair of odd and even cards selected is always a one-digit number.
   2) The positive difference of the two odd cards and the two even cards must not be the same.

   a) How many different 4-digit numbers can Alam form that satisfies the conditions above?
   b) How many different 4-digit numbers can Alam form if the product in rule 1) is a two-digit number that is less than 20?
   c) How many different 4-digit numbers can Alam form if the product in rule 1) is a multiple of 3 that is less than 40?